A Floating Vertical TEC Ionosphere Delay Correction Algorithm for Single Frequency GPS Receivers

Jade Morton, Qihou Zhou, Mathew Cosgrove
Miami University

BIOGRAPHIES

Dr. Jade Morton is an Associate Professor in the Department of Electrical and Computer Engineering at Miami University. Her research interests are digital signal processing, software GPS receivers, and modeling of the ionosphere. She holds a BS in Physics from Nanjing University, China, a MS in Electrical Engineering from Case Western Reserve University, a MS in Systems Analysis from Miami University, and a PhD in Electrical Engineering from the Pennsylvania State University.

Dr. Qihou Zhou is an Associate Professor in the Department of Electrical and Computer Engineering at Miami University. His research interests are in radar remote sensing of the ionosphere, radar coding techniques, and signal processing. He holds a BS from Harbin Technical University, China, a MS in Electrical Engineering, a MS in Mathematics, and a PhD in Electrical Engineering, all from the Pennsylvania State University.

Mathew Cosgrove is an undergraduate student at Miami University. He is expected to receiver three BS degrees in Electrical Engineering, Mathematics, and Statistics, respectively, in December 2008. He is a recipient of the Institute of Navigation undergraduate scholarship and NASA Space Grant Scholarship. He has been working on navigation related research projects under the supervision of Dr. Jade Morton and Dr. Qihou Zhou since 2005.

ABSTRACT

This paper presents an innovative method to mitigate ionosphere delay error for single frequency receivers. Traditional approach to ionosphere delay correction is carried out through modeling the total electron content (TEC) along each satellite signal path. Because of the large deviation of the TEC during different times of the day, season, and solar cycle from any mean measurements, the ionosphere delay error has always been the dominant error factor in signal frequency receivers. In our method, we allow a reference vertical TEC to "float" or to remain as an unknown in the range equation. The TEC along each satellite signal path is modeled as having its own vertical TEC whose major component is the reference vertical TEC with additional contributions from the TEC spatial derivatives. The TEC spatial derivatives can be modeled using existing ionosphere models, services that provide ionosphere measurements, or even better, the ionosphere correction algorithm (ICA) coefficients which are part of the navigation message. By having the reference vertical TEC as an additional unknown in the range equations, a minimum of five satellites is required to obtain the receiver position. As a by-product of the range equation solution, we also solve for the reference vertical TEC which can be a convenient and inexpensive way to provide information on the ionosphere. The paper describes the detailed algorithm and simulation studies that demonstrate order of magnitude improvement in ionosphere error correction using this method over the conventional method.

1. INTRODUCTION

The ionosphere is the dominant factor in single frequency GPS receivers because of the large variation of the total electron content (TEC) along the satellite-receiver signal propagation path. Typical TEC values vary from 10^{16}–10^{19} electrons/m^2 columns, depending on the time of the day, season, solar activities, and receiver location. To a first order approximation, the ionosphere range delay I can be computed using the following formula [Klobuchar, 1996]:

$$I = \frac{40.3TEC}{cf^2} \quad (1)$$

where c is the speed of radio wave propagation and f is the signal carrier frequency. One convenient TEC unit of measure is 10^{16} electrons/m^2 column which is referred to as 1 TEC unit. According to (1), 1 TEC unit corresponds to a 0.163m range delay. The nearly three orders of magnitude TEC variation, therefore, contribute to a greater than 100m range measurement uncertainty.

There are several existing methods to mitigate the ionosphere delay error. The most effective method is using dual frequency receivers. Because the ionosphere is dispersive, one can eliminate the first order ionosphere delay error by combining measurements from two frequencies. The disadvantage of a dual frequency receiver is the requirement of additional hardware and processing for the second frequency channel. Klobuchar and Kunches [3] provide a nice summary of current methods for ionosphere correction with single frequency GPS receivers. The simplest approach employed by most single frequency receivers is using the broadcast Ionospheric Correction Algorithm (ICA). The ICA relies on eight coefficients to compute the ionosphere delay correction. The coefficients are part of the satellite ephemeris message and they are typically updated once every 10 days by the GPS Master Control Facility. The ICA correction can correct about 50% of the ionosphere delay error. More complicated models such as the International Reference Ionosphere (IRI) and the Bent model allow a receiver to incorporate hundreds of coefficients to obtain the monthly average ionosphere TEC. Use of these complicated models improves the delay correction to about 75%. Further improvement in error correction can be obtained in some regions of the world where the Space-Based Augmentation System (SBAS) provides near-real time ionosphere range delay corrections from geostationary satellites. The delay data are obtained from a network of reference stations equipped with dual frequency receivers. The SBAS correction typically provides vertical ionosphere delays for a 5^0 by 5^0 grid and may resolve up to 90% of the range error over the coverage region. A GPS receiver must have the capability to receive the SBAS message to utilize the corrections.

This paper introduces an ionosphere correction scheme for a single frequency receiver for which there is no additional hardware requirement. In this scheme, we allow the absolute vertical TEC value at the observation location to "float" in the range equation as an unknown, similar to the way the receiver clock error was treated. This vertical TEC is solved along with the receiver coordinates and clock error using a minimum of five satellites' pseudorange measurements. The vertical TEC spatial derivatives over the direct satellite viewing region are modeled or derived from ICA. An iterative approach is used to first solve for an approximate receiver location without taking into consideration the ionosphere delay or clock error. This approximated solution is then used as the basis for calculating the direct satellite viewing area, the vertical TEC spatial derivatives over the area, and satellite signal direction of arrival. A refined receiver position is solved by including the ionosphere delay for each satellite-receiver path based on the receiver local vertical TEC and the TEC spatial derivatives. This scheme can be easily validated and evaluated using both simulation and real GPS pseudorange measurements.

Details of our ionosphere correction algorithm will be presented in Section 2 of this paper. Section 3 analyzes ionosphere TEC data obtained from the International GNSS Service (IGS) to support the initial validation our proposed method. The algorithm performance evaluation is shown in Section 4 using simulation for an example location in Oxford, OH (latitude: 39°30'3.390", longitude: W84°45'55.986", altitude: 262.298 m). Section 5 summarizes the current work and highlights planned relevant future investigations.

2. IONOSPHERE DELAY CORRECTION ALGORITHM

The ranging equation for a satellite-receiver pair typically takes the form:

$$\rho = r + c\delta t_r - c\delta t_s + I + T + M + \varepsilon \qquad (1)$$

$$r = \left|\vec{r}_s - \vec{r}_r\right| \qquad (2)$$

where ρ is the receiver-satellite pseudorange derived from the receiver code tracking loop, r is the true receiver-satellite range, $\vec{r}_s$ and $\vec{r}_r$ are the satellite and receiver position vectors in 3D space, δt_r and δt_s are the receiver and satellite clock errors, respectively, I and T are the ionosphere and troposphere delays, M is the multipath error, and ε lumps the receiver noise and other error factors, such as satellite orbit perturbation. In this paper, our main focus is the mitigation of ionosphere errors. Therefore, we will combine the troposphere delay, satellite clock error, multipath with the receiver noise and other errors into a single term and denote it as ε,

$$\rho = r + c\delta t_r + I + \varepsilon \qquad (3)$$

The traditional single frequency receiver approach requires pseudorange measurements from a minimum of four satellites to solve for the receiver position vector alone with an unknown receiver clock error. The ionosphere delay is typically done through modeling. The simple ionosphere correction algorithm (ICA) commonly used in a single frequency receiver relies on eight coefficients transmitted as part of the GPS navigation message to describe the worldwide behavior of the Earth's ionosphere [2]. The ICA requires the receiver's approximate latitude, longitude, observation time, and each satellite signal direction of arrival (elevation and azimuth angles). This simple algorithm gives about 50% ionosphere correction. More complicated models can at most achieve 75% correction because of the day-to-day variability of the ionosphere that cannot be captured by any model.

There have been attempts to measure the ionosphere TEC using a single-frequency GPS receiver. Cohen et al [1] uses the L1 carrier-aided code minus the L1 carrier phase changes during a GPS pass. This method cannot determine the absolute TEC, and therefore cannot correct the ionosphere delay in an ionosphere model-independent manner.

The method presented in this paper does not attempt to model the ionosphere TEC. Instead, the TEC value I for any satellite-receiver path at a given observation location takes the following approximation form:

$$I = \frac{I_v}{\cos\delta} \tag{4}$$

$$I_v = I_0 + \frac{\partial I_v}{\partial \lambda}\Delta\lambda + \frac{\partial I_v}{\partial \phi}\Delta\phi + O(I_v) \tag{5}$$

where I_v is the vertical TEC value at the ionosphere piercing point along the satellite-receiver path, δ is the slant angle at the piercing point as shown in Fig. 1, λ and ϕ are the piercing point longitude and latitude, respectively. $\Delta\lambda$ and $\Delta\phi$ are the longitude and latitude difference between the piercing point and the receiver position. The last term in (5) represents higher order spatial derivatives which we will ignore in this paper.

Substituting (4) and (5) into (3), the range equation becomes:

$$\rho = r + c\delta t_r + \frac{I_0 + \frac{\partial I_v}{\partial \lambda}\Delta\lambda + \frac{\partial I_v}{\partial \phi}\Delta\phi}{\cos\delta} + \varepsilon \tag{6}$$

In Equation (6) we have five unknowns: the receiver position vector, receiver clock error, and the vertical TEC I_0 at the receiver. The vertical TEC spatial derivatives along longitude and latitude can be obtained from models or derived from the ionosphere correction algorithm. For each satellite-receiver pseudorange measurement, δ, $\Delta\lambda$, and $\Delta\phi$ are computed for the approximate receiver position which can be obtained by solving the range equations without taking into consideration the ionosphere delay and other errors.

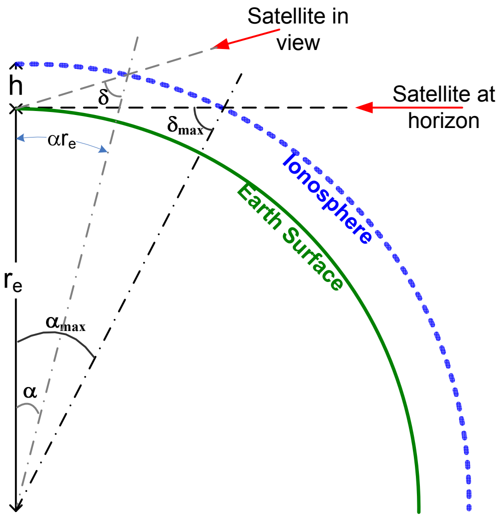

Fig. 1. Geometric relationship among GPS signal propagation path, piercing point, and direct satellite viewing area.

In essence, the method presented here allows the local vertical TEC to float as an unknown in the range equation, thereby avoiding the impossible task of modeling the large day-to-day variation and uncertainty of the ionosphere TEC. The TEC variation from different satellite to the receiver is modeled as the first order spatial derivatives of the local vertical TEC. The method will be an improvement to the current receiver implementation if the ionosphere delay error associated with the first order spatial derivative is smaller than the uncertainty of the slant TEC. Section 3 utilizes the International GNSS Service (IGS) data to demonstrate that this is indeed the case.

3. IGS IONOSPHERE DATA ANALYSIS

The IGS provides ionosphere vertical TEC values for a 5^0 x 5^0 global grid derived from a global network of monitoring stations equipped with dual frequency receivers. Fig. 2 is a typical snapshot of the global vertical TEC plot constructed using the IGS data (January 1st, 2007). The TEC scale is measured in 0.1 TEC units. Our goal here is to demonstrate that at a given location and a given time of a day, the absolute TEC variations is order of magnitude larger than the TEC variations associated with TEC spatial derivatives.

We took two entire years of IGS TEC data at Oxford, OH: year 2001 which was during the solar maximum period and year 2006 which was a quiet year. We computed the mean and standard deviation of TEC and TEC derivative along latitude and longitude using each entire year of data at every two local time increment. Figures 3 and 4 plot our computation results.

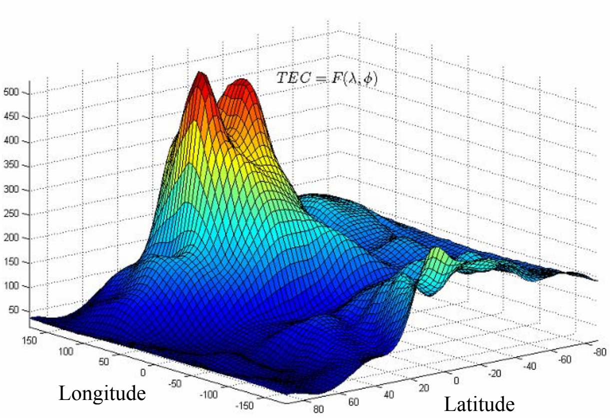

Fig.2 Global vertical TEC map constructed using IGS data for January 1^{st}, 2007.

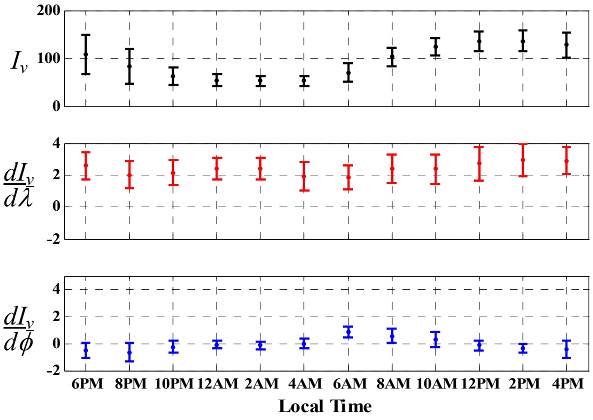

Fig.3 Annual mean and 1σ vertical TEC and vertical TEC (all in units of 0.1 TEC unit) latitude and longitude derivatives above Oxford, OH in 2006.

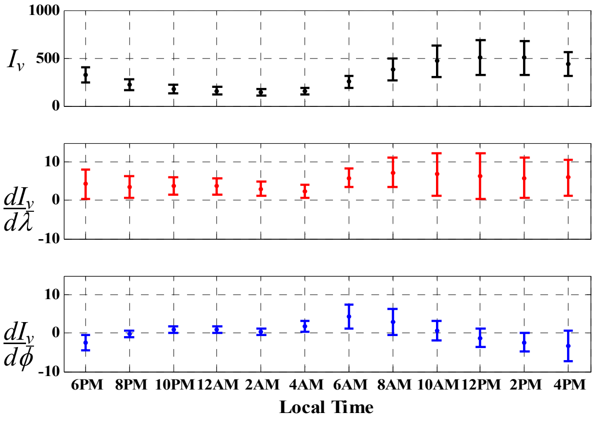

Fig.4 Annual mean and 1σ vertical TEC and vertical TEC (all in units of 0.1 TEC unit) latitude and longitude derivatives above Oxford, OH in 2001.

It is evident from Fig. 3 and 4 that the annual mean vertical TEC can be an order of magnitude different from one year to another. The standard deviation of the annual mean vertical TEC is about 10 TEC units during a relatively quiet year (2006) and about 50 TEC units during an active year (2001). In order to effectively compare these figures with the first order vertical TEC spatial derivatives, we have to estimate the satellite latitude and longitude displacement from the receiver location. Fig. 1 shows the schematic to obtain the estimation. The largest latitude and longitude offset comes from satellites at horizon. Simple geometry indicates that the largest signal path slant angle $\delta_{\max}$ at the average ionosphere height of 350 km altitude is about 71.5^0. This corresponds to maximum latitude and longitude offset α_{max} of 18.5^0 and is reduced to about 11^0 if we set an elevation mask angle of 10^0 for the direct sky viewing area.

Referring to Fig. 3 and 4, we see that the largest standard deviation from the mean vertical TEC spatial derivative was about 0.2 TEC units in 2006 and 0.5 TEC units in 2001. This is equivalent to the worst mean variations of about 2 and 5 TEC units contribution to total TEC between the two years. This variation range is about one order of magnitude smaller than the mean absolute vertical TEC variation for the same two years.

Although the above conclusion is derived from the annual vertical TEC average and standard deviation for a specific location in the mid-latitude region, the argument will hold for the shorter term mean and standard deviation. For equatorial and high latitude areas where there is more TEC variation uncertainty, the first order spatial derivatives will have a larger deviation from their mean. What should be emphasized, however, is that as long as the fluctuation in the TEC associated with first order spatial derivatives is less than that of the absolute TEC value itself, our method will have improved range measurements. It is straightforward to evaluate the variation of absolute TEC and spatial derivative contributions and using the same IGS data, and this is planned as an immediate follow up work.

There are two alternative approaches for further reducing the ionosphere delay error to improve the range solutions. One approach is to allow the first order TEC spatial derivative to "float" or become variables in the range equation. Pseudorange measurements from a minimum of seven satellites can then be used to solve for the receiver location, receiver clock error, local vertical TEC, and mean local vertical TEC latitude and longitude derivatives. The second approach is to divide the direct sky viewing area into sub-areas. Each area will have its own reference vertical TEC value. One additional satellite pseudorange measurement will be needed for each additional sub-area. For example, if the sky is divided into four sub-areas, a total of eight satellites will be needed to solve for the receiver location, clock error, and the four TEC values.

In Section 4, the IGS data will be used to simulate the receiver position solution to demonstrate the effectiveness of our algorithm in mitigating the ionosphere delay error.

4. RESULTS AND ANALYSIS

We devised the following simple way to validate the effectiveness of our algorithm. The method again utilizes the 2006 IGS database. For a given location in Oxford, OH, at a given local time, we obtain the satellite signal direction of arrival using the GPS ephemeris or almanac. We then compute ionosphere piercing point latitude and longitude offsets relative to that of the observation location ($\Delta\phi$ and $\Delta\lambda$). The first order spatial derivatives are modeled as having mean and uniform random error defined by data shown in Fig. 3. The pseudorange for each satellite is computed based on the satellite ephemeris data, local coordinate, and ionosphere TEC map over the given location and time. Clock error, troposphere delay, etc are not considered in this process. Receiver position is solved using four satellites range equations (because clock error is not included in the equation). The receiver position east and north coordinate errors computed following such a procedure are shown in Fig. 5 and 6 as the scattered red points.

These solutions are compared with the results obtained through modeling of the vertical TEC I_v itself (using three satellites), instead of the first order derivative of the vertical TEC. The results are shown in Fig. 5 and 6 as the scattered blue dots. The same pseudorange data used to generate the red dots are used here. The left figure in Fig. 5 shows an example comparison of the receiver horizontal position error obtained using the two approaches. Evidently, the horizontal position error may extend beyond 5m in north and 3m in west when the direct vertical TEC modeling approach is used. The rather compact red dot very close to the origin indicates that the method that models the spatial derivatives results in a much smaller receiver position error. The right hand side figure in Fig. 5 is a zoomed in version of the same red dot shown in the left-hand side figure. It indicates that most of the scattering is confined to within the 0.1m range. Fig. 6 shows the same comparison results obtained for January 1, 2007 at Oxford, OH in 2 hour increments. These results qualitatively demonstrated that modeling spatial derivatives can reduce the ionosphere induced position error by one or more orders of magnitude.

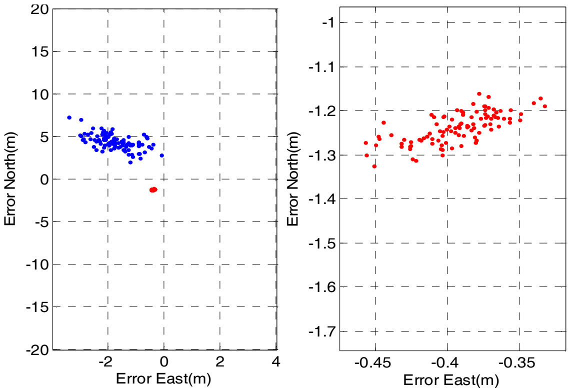

Fig. 5. Comparison of receiver horizontal position error obtained by modeling vertical TEC (blue) and modeling first order spatial derivatives of the vertical TEC (RED).

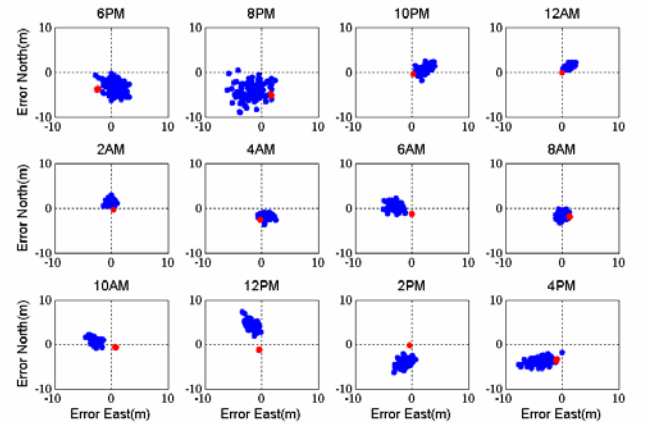

Fig. 6 Comparison of receiver horizontal position error obtained by modeling vertical TEC (blue) and modeling first order spatial derivatives of the vertical TEC (RED) at Oxford, OH for every two incremental on Jan.1, 2007.

A by-product of using Eq.(6) and modeling the spatial derivative to solve for receiver position is the local vertical TEC I_0 value. Fig. 7 plots (green dotted line) the I_0 value obtained for each of the plot shown Fig. 6. The results are compared with the vertical TEC values obtained from the IGS database for the same day, time, and location. This comparison shows that our method generates reasonable vertical TEC values.

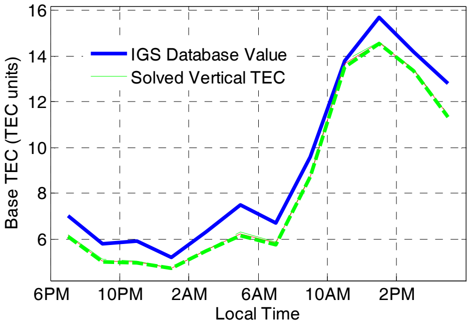

Fig.7 Vertical TEC values generated as a by-product of the range equation solution when the local vertical TEC is allowed to "float" as an unknown.

5. CONCLUSIONS AND FUTURE WORK

We presented an algorithm that avoids the direct modeling of the ionosphere TEC in the range equations. Instead, we expanded the TEC in terms of a basic reference vertical TEC and spatial derivatives in latitude and longitude. The basic reference TEC may vary drastically during different times of a day, month, year, and solar cycle and can therefore be difficult to model precisely. We allow this quantity to remain as an unknown in the range equation, similar to, but different from, the way we treat receiver clock offset. The receiver clock error is common to all satellite pseudoranges, whereas I_0 has to be corrected by the cosine slant angle factor which is different for satellites with different direction of arrivals. The burden of ionosphere modeling is transferred to the vertical TEC spatial derivatives. Our analysis using IGS database shows that the range measurement uncertainty associated with vertical TEC spatial derivatives is an order of magnitude smaller than that of the total vertical TEC. This analysis supports our belief that by modeling spatial derivatives, we can improve the ionosphere error impact on receiver position results.

To validate the algorithm, we used the IGS database to construct simulated pseudorange measurements for selected dates and times for an imaginary receiver located in Oxford, OH. We created a TEC and TEC derivatives model using one year's worth of IGS data. We compared the receiver horizontal position error generated using the TEC model and the TEC derivative model. As was anticipated, over one order of magnitude of error reduction was found using the TEC derivative model. Furthermore, we compared the "float" vertical TEC solution obtained from solving the range equations. The results are very reasonable when compared with ones stored in IGS database.

A simple way to implement this algorithm in a single frequency receiver is to utilize the ICA coefficients which are part of the navigation data message. Instead of using these coefficients to compute the ionosphere TEC values for the satellite paths, we can use these coefficients and the ICA model to compute the TEC spatial derivatives, while allowing the absolute vertical TEC to "float" or remain unknown in the range equation. This is part of our immediate future work plan. A second item in our future work plan is to use dual frequency receiver measurements to validate the vertical TEC solution obtained by our algorithm.

Additional future work includes assessing the effects of using additional satellite pseudorange measurements to (1) allow for the "floating" of the first order spatial derivatives, and (2) allow partitioning of direct satellite viewing areas into sub-areas, thereby introducing multiple unknown vertical TEC's, each associated with a sub-area. We are particularly interested in evaluating the performance of the method for equatorial and polar regions, where there are larger TEC uncertainties, and for solar active times.

REFERENCES:

[1] Cohen, C. E., B. Pervan, B. W. Parkinson, "Estimation of absolute ionospheric delay exclusively through single-frequency GPS measurements," 1992 ION-GPS Meeting, Albuquerque, Sept. 1992.

[2] Klobuchar, J. A., "Ionospheric effects on GPS," in Global Positioning Systems: Theory and Applications, edited by B. W. Parkinson, AIAA. Ch12, p485-514, 1996.

[3] Klobuchar, J. A. and J. M. Kunches, "Comparative range delay and variability of the earth's troposphere and the ionosphere," *GPS Solutions*, DOI 10.1007/s10291-003-0047-5, vol.7, p55-58, 2003.